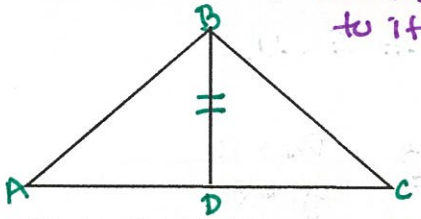
Congruent Triangle Proofs

Remember the ~~5~~ 4 shortcuts for showing that two triangles are congruent:

SSS/SAS/ASA/AAS

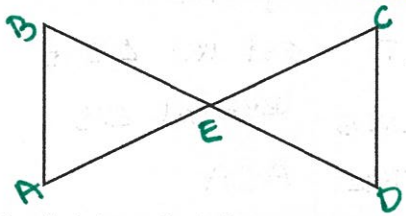
Common Properties/Theorems/ Postulates/Definitions in congruent triangle proofs:

Reflexive Property: When something is equal, or congruent, to itself



$\overline{BD} \cong \overline{BD}$ Shared side

Vertical Angle Theorem: Vertical ∠'s are ≅

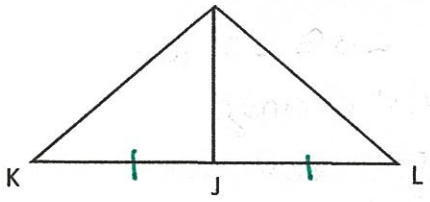


$\angle E \not\cong \angle E$ NO!

$\angle AEB \cong \angle CED$

Definition of Midpoint: Middle point - splits a segment into two congruent halves

Given: J is the midpoint of $\overline{KL}$.



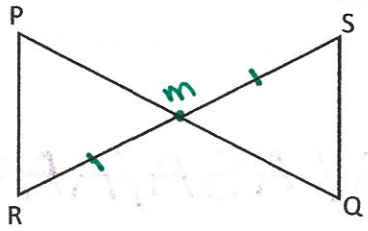
$\overline{JK} \cong \overline{JL}$

OR

$\overline{KJ} \cong \overline{LJ}$

Definition of Bisect: cut into two congruent parts

Given: $\overline{PQ}$ bisects $\overline{RS}$ → def. of bisector



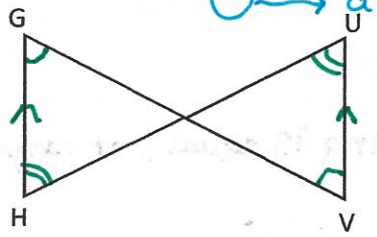
$\overline{RM} \cong \overline{SM}$

OR

$\overline{MR} \cong \overline{MS}$

Alternate Interior Angle Theorem: alt. int. ∠s are congruent

Given: $\overline{GH} \parallel \overline{UV}$ → alt. int. ∠s



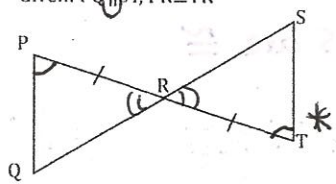
$\angle G \cong \angle V$

OR

$\angle H \cong \angle U$

Example:

Given: $\overline{PQ} \parallel \overline{ST}$, $\overline{PR} \cong \overline{TR}$ (alt. int. ∠s)



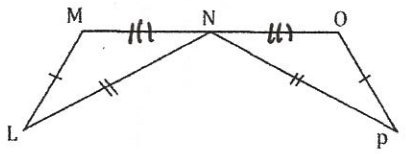
Prove: $\triangle PQR \cong \triangle TSR$

Statements	Reasons
$\overline{PQ} \parallel \overline{ST}$	given
$\overline{PR} \cong \overline{TR}$	given
$\angle P \cong \angle T$ *	alt. int. ∠'s *
$\angle PRQ \cong \angle TRS$	vertical ∠'s
$\triangle PQR \cong \triangle TSR$	ASA

*Note: you can also use $\angle Q \cong \angle S$ for alt. int. ∠'s, but that will change ASA to AAS. You do NOT have to use both!

Example:

Given: N is the midpoint (def. of midpoint) of $\overline{MO}$, $\overline{LM} \cong \overline{OP}$, and $\overline{LN} \cong \overline{PN}$

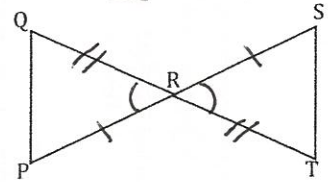


Prove: $\triangle LMN \cong \triangle PON$

Statements	Reasons
N is midpt. $\overline{MO}$	given
$\overline{LM} \cong \overline{OP}$	given
$\overline{LN} \cong \overline{PN}$	given
$\overline{MN} \cong \overline{ON}$	def. of midpoint
$\triangle LMN \cong \triangle PON$	SSS

Example:

Given: $\overline{QT}$ bisects $\overline{SP}$, $\overline{SP}$ bisects $\overline{QT}$ (def. of bisector, def. of bisector)



Prove: $\triangle QRP \cong \triangle SRT$

Statements	Reasons
$\overline{QT}$ bisects $\overline{SP}$	given
$\overline{SP}$ bisects $\overline{QT}$	given
* $\overline{PR} \cong \overline{SR}$	def. of bisector *
* $\overline{QR} \cong \overline{TR}$	def. of bisector *
$\angle QRP \cong \angle TRS$	vertical ∠'s
$\triangle QRP \cong \triangle SRT$	SAS

*Note: if it uses the vocab word twice, you MUST have it in the proof twice!